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Localization of breakage points in knotted strings

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It is a common macroscopic observation that knotted ropes under tension easily break at the knot. However, a more precise localisation of the breakage point in knotted macroscopic strings is a difficult task.

In the case of knotted spaghetti, the breakage occurs at the position with high curvature at the entry to the knot. This localisation results from joint contributions of loading, bending and friction forces into the complex process of knot breakage. The present simulations and experiments are in agreement with recent molecular dynamics simulations of a knotted polymer chain and with experiments performed on actin and DNA filaments. The strength of the knotted string is greatly reduced (down to 50%) by the presence of a knot, therefore reducing the resistance to tension of all materials containing chains of any sort.

Rock climbers know that a simple overhand knot tied on a mountaineering rope weakens it substantially. Ashley’s book of knots reports that a rope is weakest just outside the entrance of the knot. However, this description of the localisation of breakage points is hardly precise.

Recently studies of knot breaking were extended to molecular dimensions. Arai *et al* [3] demonstrated that knotted actin filaments easily break within the knotted region. Saitta *et al* [4] performed molecular modelling studies demonstrating that knotted polyethylene chains break at the entrance of the knot, whereby they precisely mapped the strain energy distribution within a knot.

Although the breaking of knotted strings is well established, there have been no dedicated studies aimed to explain the reason of this breaking. In the present study experimental breaking of knotted fishing lines or knotted cooked spaghetti was compared with numerical simulation of tight knots in order to exactly determine the breakage point.

We observe that the breakage point coincides with the point of high curvature of the string just inside the entrance to the knot. Thus, the main reason of the weakening of a knotted string is the curvature of the string. In the results section a justification of using cooked spaghetti for the experiments will be given, so that the present experiments are not so extravagant as might appear at first glance.

Friction has also another important effect on the tension within a knot. Figure 8 schematically illustrates how the friction affects the redistribution of tensile load and thus of load-induced stretching in the tightly intertwined region at the base of a knotted loop. It is apparent that before entering into the intertwined region each rope is under the total load. Upon entering the intertwined region the load becomes progressively redistributed between the two ropes due to the effect of friction. Thus in the central portion of the intertwined region each rope is only under half of the total tensile load. Since stretching caused by the load adds to that caused by the bending, and, in the case of a constant bending of the intertwined region (as that in figure 8), the rope under an increasing load will eventually break at one of the entries in to the intertwined region. In addition, the break should start on the side being stretched by bending. Therefore even if the curvature-induced stress inside the knot would exceeds the stress at the entrance, the knotted string is more likely to break close to the entrance since it is under a higher tensile load.

A recent study addressed, by molecular simulations, the influence of knotting on the strength of an individual polyethylene chain [4]. It is striking that the polyethylene chain breakage point (figures 1 and 4 in [3]) localizes in the overhand knot at the same place where we observe the breakage of the knotted spaghetti (see figure 5). This agreement between simulations performed at the atomic level and macroscopic experiments with knotted spaghetti is spectacular.