Rope running in factor 2 fall

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We consider here a factor 2 fall onto the belayer, with height $H = 2L$, where L is the length of rope from belayer to leader. The rope will stretch until its tension equals the belay device slipping threshold, and after that will run against the friction provided by that force. Conservation of energy requires that the climber's loss of potential energy be equal to the total amount of work done in stretching the rope and in feeding rope against the braking force. This energy equality provides an equation that can be solved for the amount of rope running through the belay.

The following variables are required. When, later on, a value has been used, its value is also noted.

Height of fall until belay catch $= H$	
Total fall distance $= d$	
Rope modulus $= K$	24 kN per unit relative stretch
Length of rope out $= L$	
Belay device slip force $= B$	2–6 kN
Climber weight $= W$	0.784 kN
Climber mass $= m$	80 kg
Tension in rope $= T$	
Amount of rope stretch $= s$	
Amount of rope running $= r$	
Work done in stretching the rope $= w_s$	
Work done during rope slippage $= w_r$	

By Hooke's Law, the tension in the rope is given by $T = \frac{K}{L}s$. Since the rope starts slipping at tension $T = B$, we have $B = \frac{K}{L}s$ or $s = \frac{BL}{K}$. The amount of work done in stretching a section of rope of length L by an amount s is $w_s = \frac{K}{2L}s^2$, so the amount of work done to stretch the rope until it has tension B is

$$w_s = \frac{K}{2L}\left(\frac{BL}{K}\right)^2 = \frac{B^2L}{2K}.$$

The amount of work done in feeding rope against the belay resistance B is $w_r = Br$.

The climber's loss of potential energy is Wd, where d is the total distance the climber falls and $W = mg$, with m is the climber's mass and g the gravitational constant. Adding the various causes of altitude loss gives

$$d = H + s + r = 2L + \frac{BL}{K} + r.$$

The conservation of energy equation is

$$Wd = w_s + w_r,$$

which in light of the previous observations becomes

$$W\left(2L + \frac{BL}{K} + r\right) = \frac{B^2 L}{2K} + Br.$$

Solving for r gives

$$r = \left(\frac{4KW + 2WB - B^2}{2K(B - W)}\right) L.$$

The parenthesized expression is a constant for any particular instance, so the amount of rope that will run is directly proportional to how much is out.

The braking device slipping threshold B is often quoted as being about 4 kN. I think this is potentially misleading, since there is a very large variation in belay grip strength and in the force-multiplying factors for various belay devices.

The rope modulus K for a rope with a UIAA impact rating of 9 kN can be estimated from the standard fall force equation to be about 24 kN per unit relative stretch (a unit *relative* stretch corresponds to doubling the length of the rope).

The weight W of a climber of 80 kg mass is 0.784 kN.

Here are some ball-park results for different values of B.

Braking Force (kN)	**Rope run**
2	1.3 L
3	0.7 L
4	0.4 L
5	0.3 L
6	0.2 L

From the point of view of possible slippage during a factor 2 fall, it appears that having a belay device that will provide at least 4 kN of braking force would be a significant advantage.